Physics III
ISI B.Math
Back Paper Exam : January 2, 2012

Total Marks: 100
Answer any five questions.

1. (Marks : 5 + 5 + 4 + 6)

(a) Show that if a particle of charge q and mass m moves in a time independent electric field $\mathbf{E} = -\nabla\phi(x, y, z)$ and *any* magnetic field, then the energy $\frac{1}{2}mv^2 + q\phi$ is a constant, where v is the magnitude of the velocity of the particle.

(b) Suppose the particle moves along the x-axis in the electric field $\mathbf{E} = Ae^{-\frac{t}{\tau}}\hat{\mathbf{x}}$ where A and τ are both constants. Suppose that the magnetic field is zero along the x axis and $x(0) = \dot{x}(0) = 0$. Find $x(t)$.

(c) In (b) is $\frac{1}{2}mv^2 - qxAe^{-\frac{t}{\tau}}$ a constant ? Indicate your reasoning briefly.

(d) A particle with charge q is traveling with velocity $\mathbf{v}$ parallel to a wire with uniform linear charge distribution λ per unit length. The wire also carries a uniform current I in the same direction as the velocity of the particle. What must the velocity be for the particle to travel in a straight line parallel to the wire, a distance r away?

2. (Marks : 14 + 6)

The electric potential of some charge configuration is given by the expression

$$V(r) = A\frac{e^{-\lambda r}}{r}$$

where A and λ are constants.
(a) Find the electric field and the charge density. Sketch the charge density

(b) What is the total charge Q?

3. (Marks : 10 + 10)

(a) A steady current I flows down a long cylindrical wire of radius a. Find the magnetic field both inside and outside the wire , if the the current is distributed in such a way that J is proportional to s, the distance from the axis.

(b) A long solenoid, of radius a is driven by an alternating current , so that the field inside is sinusoidal: $\mathbf{B}(t) = B_0 \cos(\omega t)\hat{\mathbf{z}}$. A circular loop of wire , of radius $\frac{a}{2}$ and resistance R, is placed inside the solenoid and coaxial with it. Find the current induced in the loop as a function of time.

4. (Marks : 8 + 12)

(a) Let the $x-y$ plane represent a grounded conducting plane. A charge $-2q$ is placed at $(0,0,d)$ and a charge q is placed at $(0,0,3d)$. Find the force on the charge $+q$.

(b) A metal sphere of radius R carrying charge q is surrounded by a thick concentric metal shell (inner radius a, outer radius b) The shell contains no net charge. Find the surface charge density σ at R, at a and at b and the potential at the centre using infinity as the reference point.

5. (Marks: 3 + 8 + 9)

(a) Why does a *sudden* unplugging of an electrical device like a toaster or an iron often result in drawing a spark?

(b) A battery of emf $\mathcal{E}$ is connected to a circuit of resistance R and inductance L. Find the current in the circuit as a function of time.

(c) Suppose we replace the resistor with a capacitor of capacitance C charged to a potential V and replace the battery by a switch. At time $t = 0$ the switch is closed. Find the current in the circuit as a function of time. How will your answer change if the resistor R is put back in series with C and L?

6. (Marks : 4 + 8 + 8) (a) Write down the full set of Maxwell's equations in differential form.

(b) Show that , for Maxwell's equations in vacuum, each Cartesian component of **E** and **B** satisfies the 3-D wave equation

$$\nabla^2 f = \frac{1}{c^2}\frac{\partial^2 f}{\partial t^2}$$

with $c = \frac{1}{\sqrt{\epsilon_0\mu_0}}$ (Note that :$\nabla \times \nabla \times \mathbf{A} = -\nabla^2\mathbf{A} + \nabla(\nabla \cdot \mathbf{A})$) Show that the waves are transverse

(c) Write down the real electric and magnetic fields for a monochromatic plane wave of amplitude E_0 , frequency ω and phase angle zero that is travelling in the negative x direction and polarized in the z-direction. Find the time average (over a cycle) of the energy density and the Poynting vector for such a wave. What does the Poynting vector represent physically ?